Professor Fabio BLANCO-MESA, PhD
Department of Economic and Administrative Sciences
School of Business Administration
Pedagogical and Technological University of Colombia
Boyacá-Colombia
E-mail: **fabio.blanco01@uptc.edu.co**
Professor Ana María GIL-LAFUENTE, PhD
E-mail: amgil@ub.edu
University of Barcelona
Barcelona-Spain

TOWARDS A COMPETITIVENESS IN THE ECONOMIC ACTIVITY IN COLOMBIA: USING MOORE'S FAMILIES AND GALOIS LATTICES IN CLUSTERING

Abstract. *The main aim of this paper is to find affinities between each Colombian region through economic activities analysis focusing on crucial role of location to be further competitiveness. The mathematical models of Moore families' and Galois lattices are used to identify a kind of industry in a region and its economic activity affinities. The results obtained from the calculation, we are shown 6 lattices formed with a great deal of affinity groupings within each of them. These groups have allowed us analysing economic activity-region by each region and identifying what kind of industry is developed within them. Finally, it is highlighted that these mathematical models give a prospective view of regional and national economic activity from general level to specific level, which can be used as tool for analyzing environments, policymaking and encouraging business development. Likewise, these models can offer a new manner to analyse socio-economic changes with a great deal with uncertainty.*
Keywords*: Decision making, Families of Moore, Galois Lattices, Clusters, Competitiveness, Colombia.*

JEL Classification: C69, O10, O44

1. Introduction

In the last decades the economic openness and globalization have simplified trade barriers making economies more or less rely on each other and affecting firms, regions and nations competitiveness. In this context, the governments have been pondering how to maintain sustainable economic growth and how to improve competitiveness. In this sense, policymakers have focused their efforts on the promotion of economic policy as a key strategy for regional development within the international economic context. In fact, competitiveness as the main objective of the regional and economic development strategy focused on the development of strengthening programme clusters (Ketels, 2013). Moreover, the geographic area is established in integrated economic areas that offer distinctive qualities for enhancing growth. Hence, the initiatives for encouraging clusters are directed towards the promotion of economic development, improving microeconomic business environment, increasing productivity and stimulating the entrepreneurial activity and the entry of new firms (Porter, 1996, 2000).

In the last years, The Republic of Colombia has shown a sustained economic growth in its economy. In Colombia have been carried out actions for making people aware of the competitiveness, strengthening institutional capacities between regional entities, and generating and disseminating of knowledge related to clusters and competitiveness (Rodríguez Delgado, 2012)as basis for regional development. However, there are still common challenges that affect social and economic development in Colombia, especially in regions that are further from strategic economic poles. Hence, the paper's main aim is to find affinities between each Colombian region by means of economic activities analysis. The Moore families' and Galois lattices are used to build a mathematical model. The mathematical application identifying a kind of industry in a region or groups of regions is developed. Likewise, it allows also grouping regions related to economic activity affinities that enable to identify localisation qualities. Thus, the paper structure is as follows: firstly, theoretical framework is concentrated on region, firm and cluster importance for competitiveness. Likewise, literature review is focused on Colombia as subject of study. Secondly, it is explained the methodological process and variables of study. Thirdly, they are presented the main results obtained and its analysis. Finally, the conclusions and implication of study are presented.

2. Theoretical framework

2.1 Competitiveness and Clusters

Globalization, economic liberalization, technological development and better management information systems have contributed to trade barriers simplification. Firms and regions have developed more effective, efficient and competitive processes to compete. This dynamic of economic activity has carried out to better use of resources available within region, which have turned into a recourse platform for firms (Snowdon and Stonehouse, 2006). In fact, an important role is played by location for business development, since it works as an operations center whereby firm and environment interact. In this context, governs have focused their efforts on the pursuit of competitiveness to sustain economic growth and prosperity. These efforts have been reflected in a comprehensive economic policy as a key factor for business and regional development within global economic context (Moncayo, 2003; Porter, 2000).

The relationship between firm and region as a source for competitiveness should be considered by approach to develop economic policy. Competitiveness is provided by the region, which has and gathers extra-organizational assets and is used by firms(Krugman, 1999). Competitiveness is developed by firms, which are assembled on the combination of income resources and are underpinned by resource-base model (Ajitabh and Momaya, 2003). Thus, firms can gain a competitive advantage within market. In addition, positive results of the firms can be spread across the region in different ways.

Nevertheless, both perspectives have a different purpose; region and firm rely heavily on each other. In fact, they are considered as a set of competences that emerge from social interaction (Lawson, 1999). On the one hand, firms compete between them within the open market and try to ensure their success through strategy (Porter, 1991). Likewise, strengths, weaknesses and external opportunities and threats of the firm are aligned on strategy (Grant, 1991; Porter, 1991). On the other hand, regions compete for improving its resource platform to operate at high levels of productivity and thus attract further foreign investment (Snowdon and Stonehouse, 2006).

Since the region and firm are mutually dependent, competitiveness can be measured by productivity of resources used within region around policies that guaranty macroeconomic stability (Snowdon and Stonehouse, 2006). According to (Mercedes Delgado, Porter, and Stern, 2012) competitiveness can be considered as the expected level of production by a person of working age given the global quality of the region as a place to do business. Likewise, Ketels (2013)asserts that the use of region’s resources for improving productivity and the quality of location for doing business are a crucial key. Hence, competitiveness is determined by the current conditions that each region offers to develop either business fabric or industry or economic sector and the use and productivity that firms make them(World Economic Forum, 2013).

Based on the above, it is clear that location as an economic space plays a key role in enhancing competitiveness. According to Ketels (2013) location is defined as a geographic area that shares an integrated economic space in which firms have access to labor market, regular supplier base and belong to the same knowledge spillovers and other kind of links. Likewise, Porter (2003) states that inside these economic spaces links by different kinds of externalities are generated. In these economic spaces there are stronger links established between several actors that promote the creation of productive grouping, which are called clusters.

According to Porter (2003) and Ketels, (2013)in a cluster there can be identified three kinds of industries by its geographical footprint. The first one is found in all regions in a similar intensity, which is called “local industries”. The second is found in certain regions with an economic activity highly concentrated, which is called “traded industries”. The third one is focused on natural resources and industries are located where resources are deposited. Thus, according to the kinds of industry developed within a cluster the stage of development of the region can be established and it can be also provided a description of environmental characteristics that affect competitiveness and productivity.

Nowadays, policies for improving business environment are based on cluster strategic, in which it is entailed economic role of the cities and economic agglomerations (Nathan and Overman, 2013). Likewise, externalities, links, side effects and support of government institutions are considered in this process (Porter, 2000).Hence, cluster strengthening has become a crucial factor to be more competitiveness. In addition, the development of the territories and business network are increased(Mercedes Delgado et al., 2012; Helmsing, 2001; Porter, 1991, 1998, 2000, 2003).

2.2 Competitiveness in Colombia

Competitiveness initiatives in Latin-American economies have been promoted from structural change. This change is supported on a variation of policies public approach with a higher coherence, articulation and coordination between sectorial and infrastructure policies, and services support. Furthermore, the importance of the regions and promotion of SME’s in different productive sectors is emphasized(OECD/ECLAC, 2012). However, the region has still low productivity, which is generated by weakly functioning of institutions, poor infrastructure and inefficient allocation of resources, which entails an insufficient level of competition and a great gap in terms of education, training, technology, and innovation-base (World Economic Forum, 2013).

Competitiveness initiatives in Colombia were started in the 90s with an analysis of competitiveness. In the period 1994-1998 National Competitiveness Council was created. In the period 1998-2002 Export Strategic Plan was raised. In the period 2002-2010 domestic politics to take advantage of treaties of integration was implemented (Ramírez, 2012). In addition, policies for promoting change production, business, science, and technology development and innovation have been proposed in parallel to these ones. These initiatives have developed positive macroeconomic conditions, although it has evidenced the existence of weak institutions and a considerable corruption, and insufficiencies in the transport infrastructures and education systems, and low diversification of economy as well (World Economic Forum, 2013). Besides, the region as a crucial key for competitiveness had never been taken into account in these initiatives (Ramírez, 2012). Nowadays, the policies to promote competitiveness have taken into account the region. This proposal raises a New Structural Economy (NSE), which is focused on promoting competitiveness within regions, fostering entrepreneurship and correcting vertical and horizontal failures through the use of market signals (World Economic Forum, 2013).

The territory and location are quite important to develop a NSE. According to Silva (2005)location can help to create comparative and competitive advantage and to develop local productive chains that stimulate the formation of small and medium business. A correct location creates closer links between suppliers, buyers and other organizations improving innovation and sector efficiency, which in turn have direct influence on local productivity growth (Porter, 2000). In this sense, the location selection should be analysed from possible existing relationships between nearby regions and towns, market signals and characteristics endogenous, and geographic of the region (Fujita and Thisse, 1996; Moncayo, 2003).

3. Methodology

We have used the resulting database of Regional Gross Domestic Product (RGDP) in Colombia of the period 2012-2013. This database was taken from Departmental Accounts of the National Administrative Department of Statics (DANE). Using this information, we have developed the application of a mathematical algorithm, which can be grouped by regions according to its economic affinities. These affinities are established on homogeneity between regional branches of activity economic and its existing relationship between each of the regions to obtain a constitutive structure. The affinity concept is the core of the mathematical application, which is supported on three main aspects: homogeneity, relationship and structure. The first one refers to each group is linked into the selected level. The second one expresses the need to link the elements of each of the sets by certain rules of nature, human will and so on. The third one requires the construction of a structure ordered that allows decision making (Gil Aluja, 1999).

Based on this concept, the methodological process is assembled in three steps. The first one, from initial matrix is assembled the fuzzy sub-set to transform in a Boolean matrix with a threshold$\alpha = n$; the second one, we are developed algebraic process to establish the relation of affinities using families of Moore and rectangular relationship (Gil Aluja, 1999); the third one, determine the order and structure of the affinities groupings through Galois lattices (Gil Aluja, 1999). Following each of the three mathematical processes is defined.

3.1 The fuzzy subset of threshold

From main matrix of the fuzzy relationship $\tilde{R}$, it is possible to demonstrate the range of possibilities to solve several problems of decision, provided that a threshold is established for each criterion, which expresses the degree, from which is considered to possess the required criteria (Gil Aluja, 1996). Hence, fuzzy subset of thresholds is defined:

$$[\tilde{U}] = \begin{array}{cc} C_1 & u_1 \\ C_2 & u_2 \\ C_3 & u_3 \\ \vdots & \vdots \\ C_n & u_n \end{array} \qquad u_i \in [0,1],\ i = 1, 2 \dots n$$

This fuzzy subset of thresholds enables a fuzzy relation $[\tilde{R}]$ to be converted into its Boolean matrix $[B]$, if it is established that:

If: $r_{ij} \geq u_i$then $b_{ij} = 1$; $r_{ij} < u_i$then$b_{ij} = 0 j = 1, 2 \dots m\ \ i = 1, 2 \dots n$ where b_{ij} represents the elements of Boolean matrix $[B]$.

3.2 Families of Moore

Starting from the concept of "power set" (Gil Aluja, 1999) given finite set E_1, its stronger set (power set), $\Pi(E_1)$ is designed as the set formed by all possible combination of its elements taken *1 by 1, 2 by 2, …, m by m*, If m is its cardinal. In this way, the set obtained is given by:

$$E_1 = \{a, b, c, \dots, m\}, \tag{1}$$

and set of all its parts or power set is given by:

$$\Pi(E_1), = \{\emptyset, a, b, c, \dots, m, ab, ac, bc, \dots, mm, E_1\}. \tag{2}$$

It is a family of $\Pi\,(\mathrm{E}_1)$, as $\mathrm{F}(\mathrm{E}_1)$, therefore: $\mathrm{F}(\mathrm{E}_1) \subset \Pi(\mathrm{E}_1)$, if $\mathrm{F}(\mathrm{E}_1)$ verifies: (1) (1) $\mathrm{E}_1 \subset \mathrm{F}(\mathrm{E}_1)$; (2) the intersection of the number of parts of $\Pi(E_1)$ belongs $\mathrm{F}(\mathrm{E}_1)$, belongs too $\mathrm{F}(\mathrm{E}_1)$, is defined by:

$$(A \in F(E_1), B \in F(E_1)) \Longrightarrow (A \cap B \in F(E_1)), \tag{3}$$

therefore$\mathrm{F}(\mathrm{E}_1)$ is a family of Moore.

From a family of Moore closing can be constructed. The Moore closing is a functional application, in which all elements of the subset $A \subset E_1$ are made to correspond with a MA, such as:

$$MA = \bigcap_{\mathrm{F} \in F_A(E_1)} F, \tag{4}$$

where$\mathrm{F_A}(\mathrm{E}_1)$represents the subset of the elements of $\mathrm{F_A}(\mathrm{E}_1)$that contains A and F all elements of $\mathrm{F_A}(\mathrm{E}_1)$.Note that mathematically to make a Moore closing must be satisfied by the properties of: Extensitivity: $\forall\ \mathrm{A} \in \Pi(\mathrm{E}_1){:}\ \mathrm{A} \subset \mathrm{MA}$; Idempotence: $\forall\ \mathrm{A} \in \Pi(\mathrm{E}_1){:}\ \mathrm{M(MA)} = \mathrm{MA}$; Isotony: $\forall\ \mathrm{A}, \mathrm{B} \in \Pi(\mathrm{E}_1){:}\ \mathrm{A} \subset \mathrm{B} \Longrightarrow (\mathrm{MA} \subset \mathrm{MB})$.Given the matrix form its analysis normally takes place through the $\alpha - cuts$ (different levels). Thus, a fuzzy relation $\tilde{\mathrm{R}}$ on being broken down by any system gives rise to a determined number of Boolean matrices.

From the fuzzy relationship $\tilde{\mathrm{R}}$, which is represented in a Boolean matrix B with a threshold $\alpha = \mathrm{n}$ are obtained right connection B^+ and left connection B^-.

The "connection to the right" B^{+}, the subset elements of E_1 such that for every A ∈ $\Pi(E_1)$, the B^{+} are the successors of all elements belonging to A.

$$\forall\ x \in A:\ B^{+}A = \{y \in E_1/(y, x) \in [B]\}, \quad (5)$$

where$B^{+}\emptyset = E_1$.

From its definition the following expression is given:

$$\forall\ x \in A \in \Pi(E_1):\ B^{+}A = \cap_{x\in A} B^{+}\{x\}. \quad (6)$$

The connection to the left, B^{-}, the subset elements of E_1 such that for every A ∈ Π (E_1), the B^{-} are the successors of all elements belonging to A.

$$\forall\ x \in A:\ B^{-}A = \{y \in E_1/(y, x) \in [B]\}, \quad (7)$$

where$B^{-}\emptyset = E_1$.

From its definition the following expression is given:

$$\forall\ x \in A \in \Pi(E_1):\ B^{-}A = \cap_{x\in A} B^{-}\{x\}. \quad (8)$$

Due B^{+} and B^{-} come from fuzzy relationship $\tilde{R}$, the closures of Moore $\Pi(E_1)$ are given by:

$$M^{(1)} = B^{-} \circ B^{+},\ \ M^{(2)} = B^{+} \circ B^{-}, \quad (9)$$

where +∘ is the max-min composition.

The closure subsets $\Pi(E_1)$ come from closure $M^{(1)}$ and $M^{(2)}$ are given by:

$$\Gamma(E, M^{(1)}) = \cup_{A\subset\Pi(E_1)} B^{+}A, \quad (10)$$

$$\Gamma(E, M^{(2)}) = \cup_{A\subset\Pi(E_1)} B^{-}A, \quad (11)$$

therefore:

$$\cup_{A\subset\Pi(E_1)} B^{+}A = \{A, B, C, \dots, M, AB, AC, BC, \dots, MM, E_1\}, \quad (12)$$

$$\cup_{A\subset\Pi(E_1)} B^{-}A = \{\emptyset, a, b, c, \dots, m, ab, ac, bc, \dots, mm, E_1\}. \quad (13)$$

In this phase of the process one and the same group of elements of set E_1 can include groups of different elements corresponding to E_2. This occurs if there is *always* a grouping of elements of E_2 that includes the remainder. Therefore, it is necessary to obtain B^{-}. In B^{-} the phenomenon occurs that for a same group of elements of E_2 there is several different of elements of E_1. In fact, there is a group of elements of E_1 that includes the remainder.

From fuzzy relationship $\tilde{R} \subset E_1 x E_2$ is considered as the starting out point to the rectangular relationship. With connection to the right and to the left, it is obtained Moore closing $M^{(1)} = B^{-} \circ B^{+}$ and $M^{(2)} = B^{+} \circ B^{-}$. In order to the family of closed elements corresponding to the Moore closing $M^{(1)}$and $M^{(2)}$ are given by:

$$\Gamma(\mathrm{E}_2, \mathrm{M}^{(1)}) = \{\mathrm{A}, \mathrm{B}, \mathrm{C}, \dots, \mathrm{M}, \mathrm{AB}, \mathrm{AC}, \mathrm{BC}, \dots, \mathrm{MM}, \mathrm{E}_1\} \qquad (14)$$

$$\Gamma(\mathrm{E}_1, \mathrm{M}^{(2)}) = \{\emptyset, \mathrm{a}, \mathrm{b}, \mathrm{c}, \dots, \mathrm{m}, \mathrm{ab}, \mathrm{ac}, \mathrm{bc}, \dots, \mathrm{mm}, \mathrm{E}_1\} \qquad (15)$$

The families of closed elements $\Gamma\,(E_2, M^{(1)})$ and $\Gamma(E_1, M^{(2)})$ are associated by the same cardinal:

$$\mathrm{car}.\,\Gamma(\mathrm{E}_2, \mathrm{M}^{(1)}) = \mathrm{car}.\,\Gamma(\mathrm{E}_1, \mathrm{M}^{(2)}) \qquad (16)$$

Note that these families constitute isomorphic lattices.

3.3 Galois Lattice

Having found the related groupings, it is established an order and structure of the single lattice. To each vertex of the single lattice, both the grouped elements of E_1 and E_2 are attached. Assembling the single lattice uses Galois lattice.

A Galois lattice is an algebraic structure that allows making clusters by affinities. Being $\Pi(E_1)$ and $\Pi(E_2)$ the power set of E_1 and E_2 are established the ordered relationship (Gil Aluja, 1996; 1999;) given by:

Firstly:

$$\forall\, X, X' \;\in \Pi(E_1),\;\; \forall\, Y, Y'\Pi(E_2)$$

$$((\mathrm{X}, \mathrm{Y}) \le (\mathrm{X}', \mathrm{Y}')) \Leftrightarrow (\mathrm{X} \supset \mathrm{X}', \mathrm{Y} \subset \mathrm{Y}'), \qquad (17)$$

Secondly:

$$\forall\, X, X' \;\in \Pi(E_1),\;\; \forall\, Y, Y'\Pi(E_2)$$

$$((\mathrm{X}, \mathrm{Y}) \ge (\mathrm{X}', \mathrm{Y}')) \Leftrightarrow (\mathrm{X} \supset \mathrm{X}', \mathrm{Y} \subset \mathrm{Y}'). \qquad (18)$$

3.4 Case Study

We have used resulting database of PIBR of the DANE, which are summarised in the following tables. In table 1,groups by Large Economic Groups (LEG) that includes economic activities developed in Colombia are shown. In table 2, it is shown the main matrix of Gross Domestic Product rate contributed by of each region, which is broken down per each economic activity.

Table 1. Grouping of Large Economic Groups

	Grouping Activities	**Economic Activities**
a	**Agriculture, Hunting, Forestry And Fishing**	1 Coffee Growing
		2 Cultivation of other agricultural products
		3 Animal Husbandry and hunting, including veterinary activities
		4 Forestry, logging and related activities
		5 Fishing, fish production in hatcheries and fish farms; service activities incidental to fishing
b	**Mining And Quarries**	6 Extraction of coal, lignitic coal and peat
		7 Extraction of crude petroleum and natural gas; service activities incidental to oil and gas extraction excluding surveying; extraction of uranium and thorium
		8 Mining of metal ores
		9 Extraction of non-metallic minerals
c	**Manufacturing**	10-37.Total Manufacturing
d	**Electricity, Gas And Water**	38 Production, collection and distribution of electricity
		39 Manufacture of gas; distribution of gaseous fuels through mains; supply of steam and hot water

		40 Collection, purification and distribution of water
		58 Elimination of waste and wastewater, sanitation and similar activities
e	**Construction**	41 Construction of complete buildings and parts of buildings; conditioning of buildings
		42 Construction of civil engineering works
f	**Trade, Repair, Restaurants And Hotels**	43 Trade
		44 Maintenance and repair of motor vehicles; repair of personal and household goods
		45 Hotels, restaurants, bars and the like
g	**Transport, Storage And Communication**	46 Land transport
		47 Water transport
		48 Transportation by air
		49 Supporting and auxiliary transport activities; activities of travel agencies
		50 Post and telecommunications
h	**Financial, Insurance, Estate Activities And Business Services**	51 Financial intermediation
		52 Real estate activities and rental housing
		53 Activities of business services excluding financial and real estate services
i	**Public Administration And Defence, Social Security**	54 Public administration and defence; compulsory social security
j	**Activities Of Social Services, Community And Personal**	55 Education Market
		56 Education nonmarket
		57 Health and social services market
		59 Activities of membership n.c.p .; recreation and cultural and sports activities; other service activities market
		60 Activities of membership n.c.p .; recreation and cultural and sports activities; other activities of non-market services
		61 Private households with employed

Source: Own elaboration based on DANE information and its statistics Atlas http://sige.dane.gov.co/atlasestadistico/

In table 3, it is shown regions grouped on zones, which are classified in common characteristics according to criteria established by Ministry of Industry, Trade and Tourism of Colombia (MinCIT) and Regional Competitiveness Committees (CRC).

Table 3. Grouping of Zone and Regions

	Zone		Regions
A	**Amazonia Zone**	A_1	Amazonas
		A_2	Caquetá
		A_3	Guainía
		A_4	Guaviare
		A_5	Putumayo
		A_6	Vaupés
B	**Andina Zone**	B_1	Antioquía
		B_2	Boyacá
		B_3	Bogotá D.C
		B_4	Cundinamarca
		B_5	Caldas
		B_6	Huila
		B_7	Norte de Santander
		B_8	Quindio
		B_9	Risaralda
		B_{10}	Santander
		B_{11}	Tolima
C	**Caribe Zone**	C_1	Atlántico
		C_2	Bolivar
		C_3	Cesar
		C_4	Córdoba
		C_5	La Guajira
		C_6	Magdalena
		C_7	San Andres y Providencia
		C_8	Sucre
D	**Pacifica Zone**	D_1	Cauca
		D_2	Chocó
		D_3	Nariño
		D_4	Valle del Cauca
Q	**Orinoquia Zone**	Q_1	Arauca
		Q_2	Casanare
		Q_3	Meta
		Q_4	Vichada

Source: Own elaboration based on Ministry of Industry, Trade and Tourism of Colombia (MinCIT) and Regional Competitiveness Committees (CRC) information's.

It relies basically on a proximity criterion. Based on the below, we have developed the mathematical application, which allow us identifying kind industry in a region or groups of regions is developed through economic activity affinities.

3. Results

We have shown average contribution to GDP made by each zone and region according its economic groups (see table 4 and 5):

Table 4. Matrix of fuzzy relationship between Zones and LEG

	a	b	c	d	e	f	g	h	i	j
A	0,078	0,105	0,019	0,013	0,083	0,135	0,069	0,064	0,197	0,197
B	0,101	0,061	0,119	0,039	0,098	0,117	0,065	0,151	0,064	0,106
C	0,077	0,157	0,075	0,041	0,078	0,143	0,065	0,102	0,077	0,120
D	0,101	0,105	0,092	0,025	0,073	0,115	0,051	0,138	0,091	0,144
Q	0,095	0,518	0,016	0,010	0,049	0,061	0,031	0,034	0,088	0,079

Based on the mathematical model explained above, we have assembled Boolean matrices (see table 6), which are obtained by fuzzy subset of thresholds.

Table 2. Main matrix of Regional Gross Domestic Product rate

ACT	a					b				c	d			e		f			g					h			i	j			d	j					
REG	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10_37**	**38**	**39**	**40**	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**	**61**	**SV**	**DT**	**GDP**
TC	0,8	4,2	3,0	0,6	0,5	3,2	9,9	1,7	0,4	7,4	2,1	0,2	0,3	3,7	4,5	6,0	0,7	5,2	2,0	0,0	0,7	0,5	2,7	3,2	3,7	4,0	9,7	0,9	5,9	3,2	0,3	1,8	0,3	0,5	94,0	6,0	100
AMA	0,0	0,0	0,5	1,8	9,7	0,0	0,0	0,0	0,0	1,8	1,1	0,0	0,2	0,0	0,0	12,7	0,0	7,2	0,0	0,2	4,3	0,5	5,0	5,2	1,8	0,5	21,7	0,2	12,9	4,5	0,2	2,7	0,2	0,2	95,2	4,8	100
ANT	0,6	2,9	2,1	0,1	0,0	0,0	1,7	1,4	0,4	13,3	4,1	0,2	0,6	5,0	3,3	9,3	1,2	3,1	2,7	0,1	0,3	0,4	2,1	5,4	7,9	8,4	4,4	2,2	2,4	2,2	0,6	1,6	0,2	0,8	91,2	8,8	100
ARA	0,0	6,5	6,9	0,7	0,1	0,0	63,9	0,0	0,1	1,4	0,6	0,0	0,1	0,3	1,9	1,8	0,1	1,9	0,5	0,0	0,1	0,0	1,2	1,0	0,8	0,3	4,8	0,1	2,3	0,9	0,1	0,5	0,0	0,1	98,6	1,4	100
ATL	0,0	0,3	1,7	0,0	0,0	0,0	0,0	0,0	0,3	14,2	3,9	0,6	1,0	4,9	2,7	8,0	1,8	4,0	4,6	0,0	0,6	0,5	2,2	5,1	7,7	7,7	4,9	2,2	3,1	3,3	0,9	1,6	0,1	1,5	89,6	10,4	100
BOG	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,2	9,3	1,2	0,4	0,7	4,1	2,6	10,4	1,5	2,3	2,4	0,0	0,8	0,4	3,2	10,1	15,0	8,2	7,6	3,5	1,0	1,4	0,6	2,8	0,3	0,9	90,7	9,3	100
BOL	0,0	2,5	1,6	0,1	0,5	0,0	3,1	0,6	0,3	24,2	2,1	0,2	0,3	5,2	5,9	4,1	0,6	3,7	3,0	0,1	0,6	0,8	1,5	2,0	3,0	5,5	4,5	0,7	3,1	2,5	0,3	0,7	0,2	1,0	84,5	15,5	100
BOY	0,2	9,3	5,4	0,2	0,1	1,9	11,6	0,0	0,9	13,2	4,1	0,2	0,2	3,2	2,7	6,6	0,9	2,9	4,2	0,0	0,0	0,3	2,0	1,8	2,8	3,9	5,4	1,1	3,5	2,1	0,2	1,5	0,3	0,4	93,0	7,0	100
CAL	3,8	4,6	2,3	0,5	0,1	0,0	0,0	0,8	0,5	11,8	4,4	0,4	0,6	3,8	6,9	4,4	1,4	4,3	2,8	0,0	0,0	0,9	2,4	3,8	4,9	7,9	6,7	1,1	4,0	3,1	0,5	2,7	0,4	0,4	92,3	7,7	100
CAQ	0,4	3,8	9,2	0,7	0,4	0,0	0,0	0,0	0,5	3,3	1,1	0,1	0,3	2,7	12,2	4,2	0,5	6,2	1,5	0,1	0,3	0,1	4,4	2,4	3,0	2,5	21,1	0,4	8,0	4,3	0,3	1,3	0,4	0,4	96,1	3,9	100
CAS	0,0	3,4	4,5	0,0	0,0	0,0	72,0	0,0	0,2	1,8	0,8	0,1	0,1	0,5	2,8	2,2	0,1	0,9	1,1	0,0	0,4	0,2	0,6	0,9	0,5	0,7	2,2	0,1	1,3	0,7	0,1	0,2	0,0	0,2	98,6	1,4	100
CAU	3,0	3,2	2,5	0,9	0,1	0,0	0,6	1,5	0,3	15,5	2,7	0,0	0,3	3,0	2,7	2,6	0,7	5,8	1,4	0,0	0,0	0,3	2,7	2,5	2,6	10,5	9,2	1,3	7,8	4,7	0,3	1,5	0,3	0,4	91,4	8,6	100
CES	0,5	3,7	3,8	0,1	0,0	45,7	1,6	0,0	0,2	3,1	2,4	0,3	0,2	3,3	2,0	3,9	0,3	2,8	2,2	0,0	0,4	0,4	1,2	1,5	1,7	3,0	4,8	0,8	3,5	2,2	0,2	0,6	0,1	0,7	97,3	2,7	100
CHO	0,0	4,3	2,2	4,9	0,3	0,0	0,0	36,7	0,3	1,3	1,0	0,0	0,0	1,3	3,3	1,9	0,1	5,9	0,3	0,1	0,3	0,1	2,6	1,5	1,6	0,1	11,7	0,2	10,2	3,8	0,0	0,8	0,3	0,5	97,7	2,3	100
COR	0,0	7,4	6,4	0,3	0,3	0,3	0,1	11,9	0,5	3,4	3,4	0,3	0,2	3,5	5,4	5,7	0,4	5,5	1,8	0,3	0,2	0,4	2,2	2,6	2,7	8,5	7,4	0,7	7,2	4,9	0,2	1,1	0,0	0,7	96,1	3,9	100
CUN	0,4	6,6	5,6	0,1	0,1	0,7	0,2	0,0	0,4	21,2	4,8	0,2	0,3	2,6	1,6	7,0	2,1	3,3	2,3	0,0	0,0	1,1	2,6	1,3	1,7	7,0	6,3	1,2	2,9	2,1	0,3	1,2	0,1	0,3	87,5	12,5	100
GUA	0,0	2,8	0,5	0,9	1,4	0,0	0,0	2,8	0,0	1,8	0,9	0,0	0,0	9,6	0,9	4,1	0,0	7,3	0,0	0,0	0,9	0,0	4,6	5,5	0,9	0,5	24,8	0,0	16,5	5,0	0,0	3,7	0,0	0,0	95,4	4,6	100
GUAV	0,0	4,8	1,0	0,7	0,0	0,0	0,0	0,0	1,5	2,4	1,4	0,0	0,2	0,0	12,0	9,4	0,2	7,4	1,0	0,2	0,9	0,0	4,8	3,6	1,5	0,3	23,8	0,0	10,6	4,4	0,2	2,9	0,2	0,9	96,1	3,9	100
HUI	4,6	4,9	1,7	0,1	0,6	0,0	18,5	0,1	0,7	2,9	2,6	0,3	0,2	6,7	13,4	4,8	0,8	3,6	5,2	0,0	0,1	0,7	1,8	2,1	2,6	3,0	5,6	0,8	4,0	2,0	0,2	1,0	1,0	0,4	97,0	3,0	100
LGU	0,1	0,9	2,4	0,2	0,0	54,4	3,9	0,0	0,4	0,8	3,5	0,2	0,1	3,6	3,2	0,9	0,1	3,9	1,1	0,0	0,1	0,3	1,4	1,1	1,2	0,2	5,4	0,2	4,7	2,3	0,1	0,3	0,1	0,2	97,4	2,6	100
MAG	0,7	7,2	6,5	0,4	0,1	0,0	0,0	0,0	0,5	5,3	2,3	0,5	0,4	8,3	5,3	6,7	0,8	7,4	3,3	0,0	0,6	1,3	2,4	3,2	3,8	3,4	7,6	1,5	7,1	5,2	0,4	1,9	0,1	0,6	94,5	5,5	100
MET	0,0	3,6	2,1	0,0	0,1	0,0	70,3	0,0	0,2	1,7	0,6	0,1	0,1	1,3	3,8	2,0	0,3	1,0	1,0	0,0	0,1	0,6	0,7	0,9	1,2	1,3	2,8	0,3	1,0	0,7	0,1	0,3	0,2	0,1	98,6	1,4	100
NAR	1,5	7,5	3,5	0,8	0,8	0,0	0,6	1,2	0,5	4,5	1,4	0,0	0,2	7,5	4,6	9,9	0,9	6,8	2,5	0,0	0,2	0,2	3,0	3,1	4,3	3,0	10,6	1,0	8,0	4,9	0,2	1,7	0,2	0,5	95,3	4,7	100
NSA	0,9	7,9	2,1	0,2	0,0	3,1	2,2	0,0	0,2	7,0	3,0	0,2	0,5	4,7	2,0	6,7	0,7	5,2	3,8	0,0	0,3	0,4	3,8	3,9	8,1	3,5	8,4	1,5	5,7	4,0	0,4	3,5	0,2	0,4	94,3	5,7	100
PUT	0,0	2,2	0,9	0,6	0,1	0,0	57,3	0,1	0,1	1,2	0,7	0,0	0,1	0,0	1,8	2,3	0,2	3,7	0,4	0,1	0,2	0,1	2,6	1,7	1,3	1,0	9,3	0,3	5,6	2,9	0,1	0,7	0,0	0,3	97,9	2,1	100
QUI	2,7	7,7	4,9	0,5	0,0	0,0	0,0	0,0	0,4	5,2	1,6	0,5	0,5	11,2	4,7	7,8	2,0	5,1	2,5	0,0	0,1	0,9	2,5	3,1	4,9	4,6	8,2	1,4	4,3	3,3	0,5	2,7	0,5	0,4	94,8	5,2	100
RIS	2,6	3,0	2,9	0,8	0,0	0,0	0,0	0,1	1,0	11,8	1,2	0,5	0,7	5,8	1,8	5,7	2,2	4,8	3,4	0,0	0,5	0,7	2,8	4,0	6,3	9,1	6,4	1,2	3,9	3,2	0,7	3,2	0,6	0,9	92,0	8,0	100
SAP	0,0	0,0	0,2	0,1	1,0	0,0	0,0	0,0	0,2	1,6	4,0	0,0	0,3	0,1	2,0	14,9	0,7	23,5	0,6	0,0	8,1	1,2	1,7	2,9	1,2	5,6	13,6	0,4	3,0	2,0	0,4	3,1	1,0	0,4	93,8	6,3	100
SAN	0,4	3,0	1,8	0,1	0,1	0,0	6,1	0,0	0,5	26,7	1,0	0,2	0,3	3,4	9,8	5,0	0,8	1,7	2,2	0,1	0,1	0,4	1,7	1,7	5,2	3,4	2,7	0,9	1,7	1,3	0,3	0,7	0,2	0,6	84,0	16,0	100
SUC	0,0	5,1	6,8	0,3	0,6	0,0	0,7	0,0	0,4	7,6	2,7	0,4	0,6	4,8	2,5	7,7	0,6	6,4	1,7	0,0	0,1	2,0	2,6	2,9	3,1	2,4	13,5	0,8	9,6	6,4	0,5	1,1	0,5	0,6	95,0	5,0	100
TOL	2,8	7,4	2,2	0,2	0,2	0,0	12,9	0,1	0,4	7,9	1,7	0,4	0,3	4,3	4,3	5,5	1,0	4,3	2,6	0,0	0,0	0,7	2,9	2,9	3,5	4,4	8,2	1,0	4,3	2,8	0,2	4,4	0,4	0,4	94,5	5,5	100
VAL	0,4	2,7	1,2	0,1	0,5	0,0	0,0	0,0	0,2	15,5	2,2	0,3	0,6	3,6	3,3	7,2	1,3	3,1	3,3	0,0	0,3	0,6	2,2	4,9	12,1	9,0	5,2	2,0	2,2	2,3	0,6	1,5	0,6	0,9	90,2	9,8	100
VAU	0,0	3,4	0,0	1,1	0,0	0,0	0,0	0,0	0,6	0,6	1,1	0,0	0,0	0,0	10,6	6,1	0,0	9,5	0,0	0,0	2,8	0,6	6,1	5,6	0,6	0,6	17,3	0,0	15,6	5,6	0,0	4,5	2,2	0,6	95,0	5,0	100
VIC	0,0	5,6	2,1	2,1	0,3	0,0	0,0	0,0	0,5	1,3	0,8	0,0	0,3	2,4	6,7	7,7	0,0	6,4	0,3	0,3	0,8	0,0	4,5	4,0	1,3	0,8	25,3	0,0	14,1	5,1	0,3	2,7	0,0	0,8	96,5	3,5	100

Source: DANE. ACT: Activities; REG: regions; TC: Total Colombia; AMA: Amazonas; ANT: Antioquía; ARA: Arauca; ATL: Atlántico; BOG: Bogotá; BOL: Bolivar; BOY: Boyacá; CAL: Caldas; CAQ: Caquetá; CAS: Casanare; CHO: Choco; COR: Córdoba; CUN: Cundinamarca; GUA: Guanía; GUAV: Guaviare; HUI: Huila; LGU: La Guajira; MAG: Magdalena; MET: Meta; NAR: Nariño; NSA: Norte de Santander; PUT: Putumayo; QUI: Qunidío; RIS: Risaralda; SAP: San Andrés y Providencia; SAN: Santander; SUC: Sucre; TOL: Tolima; VAL: Valle; VAU: Vuapés; VIC: Vichada; SV: Sub-Total Value Added; DT: Duties And Taxes; GDP:Gross Domestic Product

We have selected a threshold $\propto = 0{,}09$, since it corresponds to average contribution to GDP made by each region. The notations for the treatment of variables are simplified as follows: Group of zone called COLOMBIA are (A; B; C; D; Q), groups of regions are AMAZONIA (A1 to A6), ANDINA (B1 to B11), CARIBE (C1 to C8), PACIFICO (D1 to D2), ORINOQUIA (Q1 to Q4) and for LEG's are a; b; c; d; e; g; h; i; j.

Table 5. Matrix of fuzzy relationship between Regions and LEG

	a	**b**	**c**	**d**	**e**	**f**	**g**	**h**	**i**	**j**
A1	0,12	0,00	0,02	0,02	0,00	0,20	0,10	0,08	0,22	0,21
A2	0,15	0,01	0,03	0,02	0,15	0,11	0,06	0,08	0,21	0,15
A3	0,06	0,03	0,02	0,01	0,11	0,11	0,06	0,07	0,25	0,25
A4	0,07	0,02	0,02	0,02	0,12	0,17	0,07	0,05	0,24	0,19
A5	0,04	0,58	0,01	0,01	0,02	0,06	0,03	0,04	0,09	0,10
A6	0,05	0,01	0,01	0,01	0,11	0,16	0,10	0,07	0,17	0,29

B1	0,06	0,04	0,13	0,06	0,08	0,14	0,06	0,22	0,04	0,09
B2	0,15	0,14	0,13	0,05	0,06	0,10	0,07	0,09	0,05	0,09
B3	0,00	0,00	0,09	0,03	0,07	0,14	0,07	0,33	0,08	0,10
B4	0,13	0,01	0,21	0,06	0,04	0,12	0,06	0,10	0,06	0,08
B5	0,11	0,01	0,12	0,06	0,11	0,10	0,06	0,17	0,07	0,12
B6	0,12	0,19	0,03	0,03	0,20	0,09	0,08	0,08	0,06	0,09
B7	0,11	0,06	0,07	0,04	0,07	0,13	0,08	0,16	0,08	0,15
B8	0,16	0,00	0,05	0,03	0,16	0,15	0,06	0,13	0,08	0,13
B9	0,09	0,01	0,12	0,03	0,08	0,13	0,07	0,19	0,06	0,13
B10	0,05	0,07	0,27	0,02	0,13	0,08	0,05	0,10	0,03	0,05
B11	0,13	0,13	0,08	0,03	0,09	0,11	0,06	0,11	0,08	0,13

C1	0,02	0,00	0,14	0,06	0,08	0,14	0,08	0,21	0,05	0,12
C2	0,05	0,04	0,24	0,03	0,11	0,08	0,06	0,11	0,05	0,08
C3	0,08	0,48	0,03	0,03	0,05	0,07	0,04	0,06	0,05	0,08
C4	0,14	0,13	0,03	0,04	0,09	0,12	0,05	0,14	0,07	0,15
C5	0,04	0,59	0,01	0,04	0,07	0,05	0,03	0,03	0,05	0,08
C6	0,15	0,01	0,05	0,04	0,14	0,15	0,08	0,10	0,08	0,16
C7	0,01	0,00	0,02	0,05	0,02	0,39	0,12	0,10	0,14	0,10
C8	0,13	0,01	0,08	0,04	0,07	0,15	0,06	0,08	0,14	0,19

D1	0,10	0,02	0,16	0,03	0,06	0,09	0,04	0,16	0,09	0,16
D2	0,12	0,37	0,01	0,01	0,05	0,08	0,03	0,03	0,12	0,16
D3	0,14	0,02	0,05	0,02	0,12	0,18	0,06	0,10	0,11	0,16
D4	0,05	0,00	0,16	0,04	0,07	0,12	0,06	0,26	0,05	0,10

Q1	0,14	0,64	0,01	0,01	0,02	0,04	0,02	0,02	0,05	0,04
Q2	0,08	0,72	0,02	0,01	0,03	0,03	0,02	0,02	0,02	0,03
Q3	0,06	0,71	0,02	0,01	0,05	0,03	0,02	0,03	0,03	0,03
Q4	0,10	0,01	0,01	0,01	0,09	0,14	0,06	0,06	0,25	0,23

Table 6. Boolean Matrices by Zones and Regions

CO	**a**	**b**	**c**	**d**	**e**	**f**	**g**	**h**	**i**	**j**
A		1				1			1	1
B	1		1		1	1		1		1
C		1				1		1		1
D	1	1	1			1		1	1	1
Q	1	1								

AM										
A1	1					1	1		1	1
A2	1				1	1			1	1
A3					1	1			1	1
A4					1				1	1
A5		1							1	1

A6					1	1	1		1	1
AN										
B1			1			1		1		1
B2	1	1	1			1				
B3			1			1		1		1
B4	1		1			1		1		
B5	1		1		1	1		1		1
B6	1	1			1	1				1
B7	1					1		1		1
B8	1				1	1		1		1
B9	1		1			1		1		1
B10			1		1			1		
B11	1	1				1		1		1
CA										
C1			1			1		1		1
C2			1		1			1		
C3		1								
C4	1	1				1		1		1
C5		1								
C6	1				1	1		1		1
C7	1					1			1	1
C8	1					1			1	1
PA										
D1	1		1			1		1	1	1
D2	1	1							1	1
D3	1				1	1		1	1	1
D4			1			1		1		1
OR										
Q1	1	1								
Q2		1								
Q3		1								
Q4	1				1	1			1	1

The families of Moore are obtained by threshold $\propto \geq 0,09$ established and the criteria defined above (see table 7).

Table 7. Families of Moore

COLOMBIA	PACÍFICO	ORINOQUÍA
E_1,Ø	**E_1,Ø**	**E_1,Ø**
D, abcfhij	D_1, acfhij	Q_4, aefij
B, acefhij	D_3, eafhij	Q_1, ab
CD, bfhj	D_1D_3, afhij	$Q_1Q_2Q_3$, b
BD, acfhj	D_2, abij	Q_1Q_4, a
ACD, bfj	D_1D_4, cfhj	**E_2,Ø**
DE, ab	$D_1D_2D_3$, aij	
ABCD, fj	$D_1D_3D_4$, fhj	
ACDE, b	$D_1D_2D_3D_4$, j	
E_2,Ø	**E_2,Ø**	

AMAZONIA	CARIBE
E_1,Ø	**E_1,Ø**
A_1, afgi	C_4, abfhj
A_2, aefij	C_6, aefhj
A_6, efgij	C_4C_6, afhj
A_1A_2, afij	C_7C_8, afij
$A_2A_3A_6$, efij	C_1, cfhj
A_5, bij	$C_4C_6C_7C_8$, afj
$A_1A_2A_3A_6$, fij	C_2, ceh
$A_2A_3A_4A_6$, eij	$C_1C_4C_6$, fhj
$A_1A_2A_3A_4A_5A_6$, ij	C_1C_2, ch
E_2,Ø	C_2C_6, eh

	$C_1C_4C_6C_7C_8$, fj
	$C_3C_4C_5$, b
	$C_1C_2C_4C_6$, h
	E_2,Ø

ANDINA	
E_1,Ø	$B_5B_6B_8$, efj
B_5, acefhj	$B_2B_4B_5B_6B_7B_8B_9B_{11}$, af
B_5B_8, aefhj	$B_4B_5B_7B_8B_9B_{11}$, ah
B_6, abefj	$B_5B_6B_7B_8B_9B_{10}B_{11}$, aj
B_{11}, abfhj	$B_1B_2B_3B_4B_5B_9$, cf
B_5B_9, acfhj	$B_1B_3B_4B_5B_9B_{10}$, ch
B_6B_{11}, abfj	$B_5B_8B_{10}$, eh
$B_5B_7B_8B_9B_{11}$, afhj	$B_1B_3B_4B_5B_7B_8B_9B_{11}$, fh
B_5B_{10}, cefh	$B_1B_3B_5B_6B_7B_8B_9B_{11}$, fj
$B_1B_3B_5B_9$, cfhj	$B_1B_3B_5B_7B_8B_9B_{11}$, hj
$B_2B_6B_{11}$, abf	$B_1B_2B_3B_4B_5B_9B_{10}$, c
$B_2B_4B_5B_9$, acf	$B_5B_6B_8B_{10}$, e
$B_5B_6B_7B_8B_9B_{11}$, afj	$B_1B_2B_3B_4B_5B_6B_7B_8B_9B_{11}$, f
$B_1B_3B_4B_5B_9$, cfh	$B_1B_3B_4B_5B_6B_7B_8B_9B_{11}$, h
	E_2,Ø

We have made Galois lattices from each of the 6 families of Moore obtained (see figure 1 to figure 4). Each affinity relationship within lattice represents a factor of the family of Moore, which is assembled by regions that have homogeneous characteristics, i.e. common economic activities in the LEG. Lattices are assembled by several levels, which are ordered between thresholds E1 and E2. Levels are given by number of LEG grouped and ordered horizontally. Each factor of lattices is represented by a dot. Each line assembles lattices and establishes existing relationship between each of the factors. We have ordered lattices from left (E1, ∅) to right (E2, ∅) in ascending order according to the number of LEG grouped.

3.1 Analysis of Result

Via Graphs presented below, we have shown lattices formed from affinity relationship with a degree of homogeneity of 91% between each of regions and economic activities developed in Colombia. We have assembled six different graphs. Graph 1 (see fig. 1) has shown affinity relationship between each 5 zones. Graphs 2-3-4 (see fig. 2), 5(see fig. 3) and 6 (see fig. 4) have shown affinity relationship of each of the regions according to the zone is located. These lattices allow identifying and analysing clusters of economic activities related to kind of industry is developed and location characteristics. Graphs have suggested us two distinctive features of the groups and its distribution of regional economic activity. Firstly, we are noticed that the left side of the graph is shown that the zones and regions are grouped around specific economic activity. Secondly, we are noticed that the right side of the graph is shown that an amount of economic activities are clustered in a particular area on which a link of heterogeneous economic activities that drive the economic is formed. The transition from level to another one allows us to observe the evolution of clusters from a small number of regions that share a wide variety of economic activities to regions in which geographical areas are oversized and just share a few common economic activities. Graphs have shown regions clustered within limits E_1 and E_2, the number of clusters by each level and shared characteristics (see table 8). Following we will be analysed each of the graph and will be highlighted main relationship between each of them.

Table 8. Relationship established in each level

	Colombia			Orinoquía			Pacífica			Amazonía			Caribe			Andina		
	G	C	Z&R	G	C	Z&R	G	C	Z&R	G	C	Z&R	G	C	Z&R	G	C	Z&R
Level 1		E1			E1			E1			E1			E1			E1	
Level 2	1	1	4	2	1	2 and 3	1	1	4	1	2	6	2	1	3 and 4	4	1	4 to 10
Level 3	2	2	2 and 4	1	2	1	2	3	3	3	3	1 and 4	3	2	2 and 5	9	2	3 to 8
Level 4	1	3	3	1	5	1	2	4	1 and 2	2	4	2 and 3	3	3	1 and 4	5	3	3 to 6
Level 5	1	4	2		E2		1	5	2	3	5	1	3	4	1 and 2	4	4	2 to 5
Level 6	1	5	2				2	6	1		E2		2	5	1	4	5	1 and 2
Level 7	1	7	1					E2						E2		1	6	1
Level 8		E2															E2	

G: GROUPS C: CHARACTERISTICS Z&R: ZONE AND REGIONS

Graph 1 is assembled by Colombia's zone and shows us a holistic view about economic activity configuration. We have noticed that development of economic activity is focused on three LEGS': b-f-j. On the one hand, the main activity for industrial development is based on the exploitation of natural resources this occurs in four of the five zones grouped. On the other hand, the main activities for local industry development is based on trade, repair, restaurants and hotels and social service, community and personal activities although these activities are common for all regions its main intensive activity occur in zones A-B-C-D.The activities for trade industry development are highlighted B and C zones, since they cluster a large number of LEG's. Thus, economic activity configuration is focused mainly on LEG b and LEG f and j are becoming support activities to develop other types of industries. In this sense, economic activities developed in LEG j are become great importance as economic support tissue.

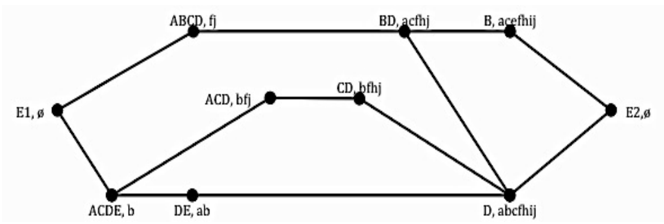

Figure 1. Graph 1. Galois lattice for Colombia Zones

Based on the above, we have analysed graphs of the specific regions focusing on groups that have not direct incidence with intensive mining activity. In figure 2 are found graphs 2-3-4 corresponding to zones of Orinoquia, Pacifico and Amazonia. Firstly, Orinoquia zone (graph 2) has shown us that its main economic activities are based specifically on LEG b and a. This implies that industrial, trade and local industry are relied directly on intensive mining activity and exploitation of natural resources. Secondly, Pacific zone (graph 3) has shown that all regions are directly related to economic activities of LEG j. In this zone are highlighted regions D_1-D_2-D_4, since grouping between them can be identified potential clusters related to trade and local industry. However, intensive mining activity is developed within this zone, in region D2 specifically. Likewise, activities of LEG i have a great presence in regions D_1-D_2-D_3, since defence of the territory activities are extremely intense by armed conflict in Colombia. Thirdly, a wilderness area is formed by Amazonia zone (graph 4). This graph has shown that all regions are directly related to economic activities of LEG j and i, which support regional development. It is noticed that local industries revolve around exploitation of natural resources as main industry, trade and transport as trade industry and social service as support of them. Likewise, defence of the territory activities are also presented.

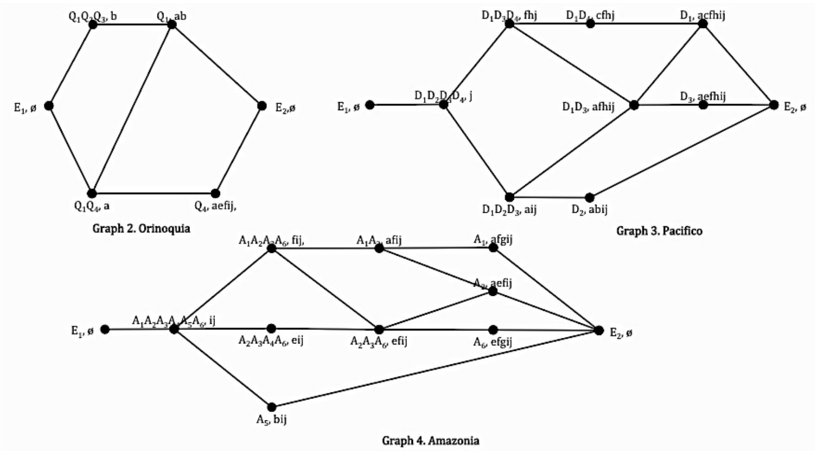

Figure 2. Graph 2-3-4. Galois Lattices for Amazonia, Orinoquia and Pacifico

Caribe zone (fig. 3 graph 5) has shown us that economic activities are focused on LEG b, f, h and j. It is noticed that industrial development is oriented towards LEG h and f and supported by j although LEG b has a strongly presence in this zone. From this activities are formed trade and local industries. Three different clusters of activity-region there are found within the graph.

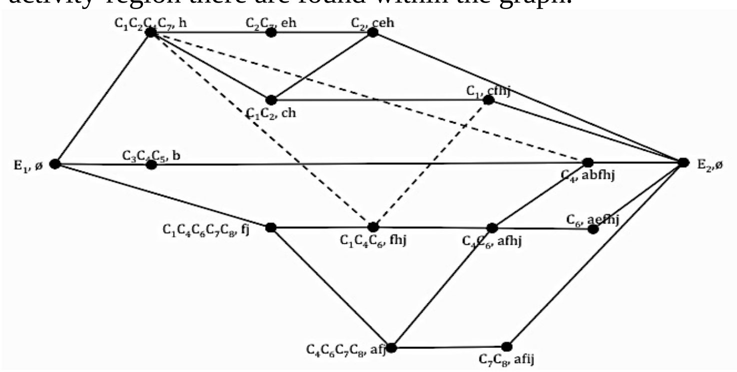

Figure 3. Graph 5. Galois lattice for Caribe zone

On the upper part of the graph are found activities related to LEG h, which have formed an economic triangle linked around activities c and e and regions C_1-C_2-C_4-C_7. On the middle part of the graph it is found that the main economic activity is b, which is focused on region C_3-C_4-C_5. On the button of the graph, it is found activities related to LEG f and j, which have also formed an economic triangle linked around activities a and h and regions C_1-C_4-C_6-C_7-C_8. Thus, from this relationship established can be made widely analyses and identified new productive proposals.

Andina zone (fig.4 graph 6) has shown is formed by regions central area. This graph has shown that economic activities are focused on LEG c, e, f and h, which encourage industrial development and support trade and local industry. Likewise, it is noticed that affinity relationships established between LEG c, f and h are further wide, intertwined and complex, which are given by a great number of regions grouped. Such complex interconnection gives further possibilities to find and detect potential clusters and economic synergies. Hence, within Andina zone either common activities grouping a great number of regions or regions that grouped a few number of activities are highlighted and needed widely analysis.

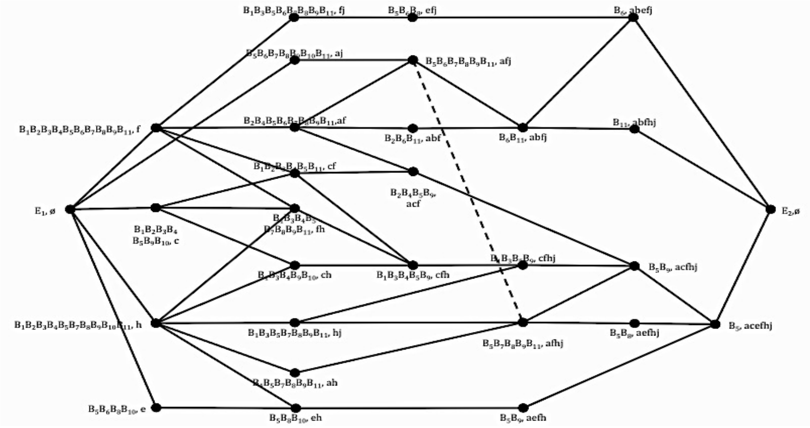

Figure 4. Graph 6. Galois lattice for Andina zone

Finally, the mathematical model has allowed to observe from a holistic view that each region has a great deal of strengths to develop its economy either individual or joint manner. The affinity relationship established has shown us that location of the regions gives specific condition and resources around each LEG. Thus, this model can aid to identify industries established within or on the limit of the region and kinds of externalities can link them. Hence, under this characterization of the regions can be identified regional capacities and key factors to be used by firms and industries.

4. Conclusions

Competitiveness of a region is assessed by productivity of its endogenous resources through key factors that drive the booster, efficiency and innovation determining level of development within economy. We have analysed competitiveness from an economic and manage perspective, emphasizing the importance of location as supplier of resources and firms managing them. We have studied clusters as agglomerations firms established within geographic space, which encourage competition and make efficient use of available resources through set of competencies that emerge from social interaction between firm and environment. Such reasoning has encouraged policymakers to develop strategies to improve competitiveness through strengthening programme clusters. With this approach, we have taken Republic of Colombia as case study.

Based on the above, we have used a mathematical model, which allows combining models for the uncertainty management, such as: Families of Moore and Galois lattices, and classical quantitative analysis for dealing data. A great number of affinities between groups of regions and economic activities highlights the results obtained. We have found grouping from general level to specific level. These groups have given a prospective view of environment in which economic activity in Colombia is developed. We have highlighted 1) that this country depends clearly on natural resources as economic base; 2) industrial, trade and local development are focused on activities related to LEG c, f and h with different intensity for each region and associated to others activities; 3) military action of defence within territory is intensive;4) transport and electricity, gas and water are not linked to any region. Furthermore, it is shown LEG j is a great deal of regions and it is needed to analyse in depth each activities and its importance as economic support tissue.

The prospective view is given by methodology applied. This methodology is different from other ones by two items. The first one, we have taken as a basis data analysis ex-post. The second one, the models of families of Moore and Galois lattices provide the prospective part. Both items can aid to correct decision making, since it is allowed forming and showing groups of holistic manner according to current affinities either economy or social and so on. Finally, we have provided a novel tool to analyse social and economic environment, which can aid policymakers and institutions to manage encouraging business development adequately within geographically heterogeneous country. Likewise, the proposed model gives the possibility to move towards new studies, in which it can be studied the level of intensity of each relationship making use of other algorithms and models.

REFERENCES

[1] **Ajitabh, A. and Momaya, K. S. (2003),** ***Competitiveness of Firms: Review of Theory****;* Frameworks and Models;
[2] **Delgado, M., Porter, M. and Stern, S. (2012),** ***Clusters, Convergence and Economic Performance*** (pp. 1–53). Cambridge;
[3] **Fujita, M. and Thisse, J.-F. (1996),** ***Economics of Agglomeration*****.** *Journal of the Japanese and International Economies*, *10*(4), 339–378;
[4] **Gil Aluja, J. (1996),** ***Towards a New Paradigm of Investment Selection in Uncertainty.*** *Fuzzy Sets and Systems*, *84*(2), 187–197;
[5] **Gil Aluja, J. (1999),** ***Elements for a Theory of Decision in Uncertainty***. Dordrecht: Kluwer Academic Publishers;
[6] **Grant, R. M. (1991),** ***Porter's "Competitive Advantage of Nations: An Assessment.*** *Strategic Management Journal*, *12*(7), 535–548;
[7] **Helmsing, B. (2001),** ***Externalities, Learning and Governance: New Perspectives on Local Economic Development.*** *Development and Change*, *32*(2), 277–308;
[8] **Ketels, C. (2013),** ***Recent Research on Competitiveness and Clusters: What Are the Implications for Regional Policy?*** *Cambridge Journal of Regions, Economy and Society*, *6*(2), 269–284;
[9] **Krugman, P. (1999),** ***The Role of Geography in Development.*** *International Regional Science Review*, *22*(2), 142–161;
[10] **Lawson, C. (1999),** ***Towards a Competence Theory of the Region*****.** *Cambridge Journal of Economics*, *23*(2), 151–166;
[11] **Moncayo, É. (2003),** ***New Theories and Conceptual Focuses on Regional Development: ¿Toward New Parading*****?***Revista de Economía Institucional*, *5*(8), 32–65;
[12] **Nathan, M. and Overman, H. (2013),** ***Agglomeration, Clusters and Industrial Policy***. *Oxford Review of Economic Policy*, *29*(2), 383–404;
[13] **OECD/ECLAC. (2012),** ***Latin-American Economy Perspectives2013: PYMES Polices forStructural Change***. Paris: OECD Publishing;
[14] **Porter, M. (1991),** ***Towards a Dynamic Theory of Strategy***. *Strategic Management Journal*, *12*(S2), 95–117;
[15] **Porter, M. (1996),** ***What is Strategy*****?** *Harvard Business Review*, *74*(6), 61–78;
[16] **Porter, M. (1998),** ***Clusters and the New Economics of Competition***. *Harvard Business Review*, *76*(6), 77–90;
[17] **Porter, M. (2000),** ***Location, Competition, and Economic Development: Local Clusters in a Global Economy*****.** *Economic Development Quarterly*, *14*(1), 15–34;
[18] **Porter, M. (2003), T*he Economic Performance of Regions.*** *Regional Studies*, *37*(6-7), 545–546;
[19] **Ramírez, J. (2012),** ***Competitiveness Regional in Colombia: Framework, Findings and Recommendations of the Competitiveness Strategy Center.*** *Observatorio de Competitividad del Centro de Estrategia y Competitividad***;**
[20] **Rodríguez Delgado, M. C. (2012),** ***Clusters Management in Colombia: A Tool for Competitiveness***. Bogotá: Ediciones Uniandes.[In Spanish];

[21] **Silva, I. (2005),** ***Local Economic Development and Regional Competitiveness. Revista de La CEPAL***, *85,* 81–99.[In Spanish];
[22] **Snowdon, B. and Stonehouse, G. (2006),** ***Competitiveness in a Globalised World: Michael Porter on the Microeconomic Foundations of the Competitiveness of Nations, Regions and Firms.*** *Journal of International Business Studies*, *37*, 163–175;
[23] **World Economic Forum (WEF) (2013).** ***The Global Competitiveness Report 2013-2014.World Economic Forum***. Geneva. Retrieved from http://reports.weforum.org/the-global-competitiveness-report-2013-2014/.